Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following:

i) $\lim_{x \to 0} (1+x)^{1/x} = ?$

   a) 1   b) 0   c) $\dfrac{2}{3}$ ✓d) e

ii) If α, β, χ be the roots of the equation $x + yn = 2$ then $\sum x^2 =$

   a) 0   b) 14   c) –14   d) 4   **[wrong question]**

iii) An element x in a ring R is zero divisor if

   a) $x \cdot b = 0$ ✓b) $x \cdot b = 0$, for some non zero element b in R

   c) $x \cdot b \neq 0$, for all element b in R d) none of these

iv) The value of $\int\limits_{-1}^{2} |x| \, dx$ is

a) 3   b) 5   ✓c) $\dfrac{5}{2}$ d) 0

v) The value of $\dfrac{d}{dx}(\log_e x)$ is equals to

✓a) $\dfrac{1}{x}$ b) $\log\left(\dfrac{1}{x}\right)$ c) $\left(\dfrac{1}{n}\right)\log_a e$ d) $a \log e$

vi) If $A = \{2, 4, 6\}$ and $B = \{1, 3, 5, 7\}$, then $A \cup B$ is

a) $\{0\}$ ✓b) $\{1, 2, 3, 4, 5, 6, 7\}$ c) $\{1, 2, 4, 5, 6, 7\}$ d) $\{0, 2\}$

vii) If A is a square matrix then

  √a) $A + A^T$ is symmetric   b) $A + A^T$ is skew symmetric

  c) $A - A^T$ is symmetric   ✓d) $A - A^T$ is skew symmetric

viii) The matrix $A = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$ is on

  ✓a) orthogonal matrix   b) idempotent matrix   c) identity matrix   d) none of these

ix) If $y = 2at$ and $x = at^2$, then $\dfrac{dy}{dx}$ at $t = 1$ is

  ✓a) 1   b) $2a$ c) -1 d) $2a^2$

x) The polar form of the equation $x^2 + y^2 - 8y = 0$ is

  a) $r = 8\cos\theta$ ✓b) $r = 8\sin\theta$ c) $r^2 = 8\cos\theta$ d) none of these

xi) If $A = \{1, 2, 3, 4, 8\}$, $B = \{2, 4, 6, 7\}$ then $A \Delta B$ is

  a) $\{2, 4\}$ b) $\{1, 2, 3, 4, 6, 7, 8\}$ c) ϕ ✓d) $\{1, 3, 6, 7, 8\}$

xii) The diagonal elements of a real skew-symmetric matrix are

  a) 1   b) -1 c) 2   ✓d) 0

Group – B
(Short Answer Type Questions)

2. A function $f(x)$ is defined as follows

$$f(x) = x^2 \qquad \text{when } 0 < x < 1$$
$$= x \qquad \text{when } 1 \le x \le 2$$
$$= 2 - x \qquad \text{when } 2 \le x < 3.$$

Show that the $f(x)$ is continuous at $x = 2$.

See Topic: **LIMIT, CONTINUITY & DIFFERENTIABILITY**, Short Answer Type Question No. 8.

3. Evaluate $\displaystyle\int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$.

See Topic: **DEFINITE INTEGRALS**, Short Answer Type Question No. 6.

4. If α, β, γ be the roots of the cubic $x^3 + px + q = 0$, then find the equation whose roots are $\dfrac{\beta + \gamma}{\alpha^2}, \dfrac{\gamma + \alpha}{\beta^2}, \dfrac{\alpha + \beta}{\gamma^2}$.

See Topic: **POLYNOMIAL**, Short Answer Type Question No. 15.

5. Prove that the ring of matrices of the form $\begin{bmatrix} x & y \\ -y & x \end{bmatrix}$ of real number is a field.

See Topic: **MATRICES**, Short Answer Type Question No. 9.

6. In a survey concerning the smoking habits of consumers it was found that 55% smoke cigarette-A, 50% smoke cigarette-B, 42% smoke cigarette-C, 28% smoke cigarette-A & B, 20% smoke cigarette-A & C, 12% smoke cigarette-B & C and 10% smoke all the three cigarette. What percentage do not smoke?

See Topic: **SET THEORY**, Short Answer Type Question No. 4.

Group – C
(Long Answer Type Questions)

7. a) If $y = \sin(m \sin^{-1} x)$, then show that $(1 - x^2) y_{n+2} - (2n + 1) x y_{n+1} + (m^2 - n^2) y_n = 0$.

b) If α, β, γ are the 3 roots of $x^3 + px^2 + qx + r = 0$ obtain the value of $\sum (\alpha - \beta)^2$.

c) Evaluate $\int \frac{1}{x^2} e^{1/x} dx$.

a) See Topic: SUCCESSIVE DIFFERENTIATION, Long Answer Type Question No. 4.

b) See Topic: POLYNOMIAL, Long Answer Type Question No. 6.

c) See Topic: INDEFINITE INTEGRATIONS, Short Answer Type Question No. 7.

8. a) If $u = \frac{y}{z} + \frac{z}{x} + \frac{z}{y}$ then prove that, $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$.

b) If by a rotation of rectangular co-ordinate axes without change of origin expressions $ax + by$ and $cx + dy$ are transformed into $a'x' + b'y'$ and $c'x' + d'y'$. Show that $a'd' - b'c' = ad - bc$.

c) Reduce the following equation to its canonical form and determine the nature of the conic represented by it: $3x^2 - 8xy - 3y^2 + 10x - 13y + 18 = 0$

a) See Topic: FUNCTION OF SEVERAL VARIABLES, Long Answer Type Question No. 4.

b) See Topic: TRANSFORMATION OF CO-ORDINATES, Long Answer Type Question No. 3.

c) See Topic: GENERAL EQUATION OF SECOND DEGREE, Long Answer Type Question No. 3.

9. a) Evaluate: $\lim\limits_{n \to \infty}\left[\frac{n}{n^2 + 1^2} + \frac{n}{n^2 + 2^2} + \ldots\ldots + \frac{n}{n^2 + n^2}\right]$.

b) Using mean value theorem prove the following inequality:

$$x < \sin^{-1} x < \frac{x}{\sqrt{1 - x^2}}, \text{ if } 0 < x < 1$$

c) Expand $\sin x$ in power of x in infinite series.

a) See Topic: LIMIT, CONTINUITY & DIFFERENTIABILITY, Short Answer Type Question No. 9.

b) See Topic: FUNCTION OF SEVERAL VARIABLES, Short Answer Type Question No. 3(a).

c) See Topic: FUNCTION OF SEVERAL VARIABLES, Short Answer Type Question No. 3(b).

10. a) Solve the equation by Cardan's method: $2x^3 + 3x^2 + 3x + 1$

b) Evaluate: $\int \frac{x^2 dx}{(x^2 + a^2)(x^2 + b^2)}$

c) If $y = x^{x-1} \log x$, show that $y_x = \frac{(x-1)!}{x}$.

a) See Topic: POLYNOMIAL, Long Answer Type Question No. 8.

b) See Topic: INDEFINITE INTEGRATIONS, Short Answer Type Question No. 9.

c) See Topic: MISCELLANEOUS, Short Answer Type Question No. 4.

11. a) Prove that $|A \cup B| = |A| + |B| - |A \cap B|$ where A and B are two non-empty sets.

b) If $A = \{a, b, c, d\}$ $B = \{b, c, p, q\}$, then find out $A \times B$, $B \times A$ and $A \Delta B$.

c) Define power set. Find the power set of $\{a, b, c\}$.

See Topic: SET THEORY, Long Answer Type Question No. 7.